STATE FEEDBACK CONTROL AT PERISCOPE DEPTH

INTRODUCTION

State feedback control

One popular means of control is to feed back the system states after the application of linear gains. System response of linear systems subjected to this type of control is predictable, and a variety of tools are available for control law gain selection.

The ship's control party on a submarine with conventional indications does not have the full state of the ship to operate from. Although the actual instrumentation may vary somewhat, in general a few analog indications are used in conjunction with a digital depth indication. For this reason, various levels of partial state feedback were used to evaluate the effects of missing indications.

The use of different state feedback schemes was felt to be appropriate to model human operators. The treatment of airplane pilots as a control law "has come to be recognized as a quasilinear element for random-appearing tracking tasks related to piloting. At the same time, the pilot retains spectacular nonlinear gain changing, mode switching, and goal seeking precognitive control capabilities as yet only partially explored." (Graham and McRuer, 1991, p. 1093) In this context, it was assumed submarine "pilots" could be treated in a similar fashion, with feedback from each operating state determined with linear gains.

The use of a first order lag was considered to model the combined human and control surface response time. It was found that reasonable lag values (on the order of a half second) had minimal effect on the control response and corresponding submarine motions. Because of the computational expense, the control response time was neglected.

State feedback control of the linear system

$$\dot{x} = Ax + Bu \tag{69}$$

where:

$A \in \Re^{mxm}$, state matrix

$B \in \Re^{mxn}$, control matrix

$$x \in \Re^{mx1} \text{, state vector}$$

$$u \in \Re^{nx1} \text{, control vector}$$

can be expressed as:

$$u = Kx \tag{70}$$

where:

$$K \in \Re^{nxm} \tag{71}$$

The system given by Equation (69) subject to the control law given by Equation (70) has the following closed loop dynamics matrix:

$$A_c = (A + BK) \tag{72}$$

The eigenvalues of the closed loop dynamics matrix will be related to the system stability and responsiveness. In general, the real portion of the eigenvalues must be negative for system stability. Also the more negative the eigenvalues, the faster the system response.

SUBOFF simulation parameters

Wave force data was available for the SUBOFF for four different cases. These were sea states three and four with head and beam directions. All were valid at a speed of six knots and depths greater than fifty feet.

At six knots, the linear state representation used for eigenvalue determination and control law design is:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -0.0179 & 3.7101 & 0.0196 & 0 \\ 0.0006 & -0.0680 & -0.0034 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -10.1269 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix} + \begin{bmatrix} -0.0628 & -0.1009 \\ 0.009 & -.0027 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_b \\ \delta_s \end{bmatrix} + \begin{bmatrix} F_d(t) \\ M_d(t) \\ 0 \\ 0 \end{bmatrix} \tag{73}$$

$$F_d(t) = F_{trim} + F_{wave}(t)$$

$$M_d(t) = M_{trim} + M_{wave}(t)$$

All simulations were performed using the nonlinear equations of submarine dive plane motion:

$$\dot{w} = a_{11}uw + a_{12}uq + a_{13}\sin(\theta) + b_{11}u^2\delta_b + b_{12}u^2\delta_s + F_d\cos(\theta) + e_{11}q^2 + e_{12}qw \tag{74}$$

$$\dot{q} = a_{21}uw + a_{22}uq + a_{23}\sin(\theta) + b_{21}u^2\delta_b + b_{22}u^2\delta_s + M_d\cos(\theta) + e_{21}q^2 + e_{22}qw \tag{75}$$

$$\dot{\theta} = q \tag{76}$$

$$\dot{z} = w\cos(\theta) - u\sin(\theta) \tag{77}$$

$$\dot{x} = w\sin(\theta) + u\cos(\theta) \tag{78}$$

The simulations were performed using a commanded depth of 55 feet and using a zero error initial state vector. Commanded pitch angle, heave and pitch rate were all zero. The depth was chosen to provide a good representation of actual submarine periscope operating depth.

State feedback implementation with SIMULINK®

The state feedback controller was implemented in the SIMULINK® model shown in Figure 11. This block was designed to use an optional feedforward signal, and also to facilitate the use of integral control (Both feedforward and integral control are discussed later in this chapter). Deflection limits were placed on the control surfaces. Control surface rate limits were not included, but could be easily added. These limits are of interest because of the relationships between control surface rates, hydraulic plant size requirements and noise from control surface operations.

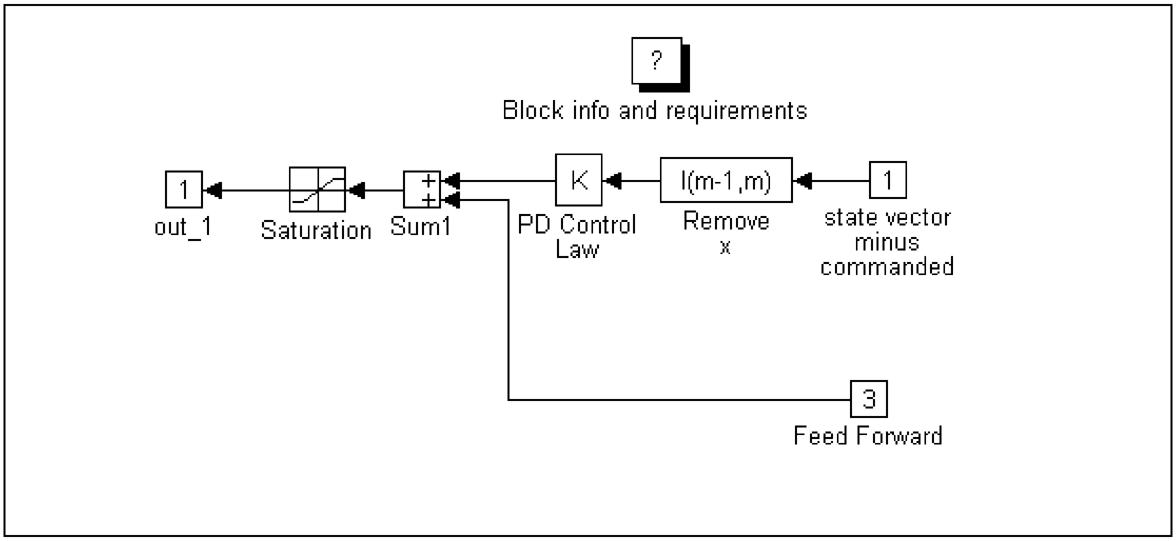

Figure 11. State feedback control block diagram

A SIMULINK® model was developed to incorporate the submarine dynamics of Chapter II, the wave forces of Chapter III, and the state feedback control law. Also included was a logical means of adjusting the submarine's trim. This was done by adding ballast in units of thousands of pounds at the center of buoyancy, and shifting ballast from the forward trim tank to the after trim tank in units of thousands of pounds. The details of the trim model are shown in Figure 12, while the overall model is shown in Figure 13.

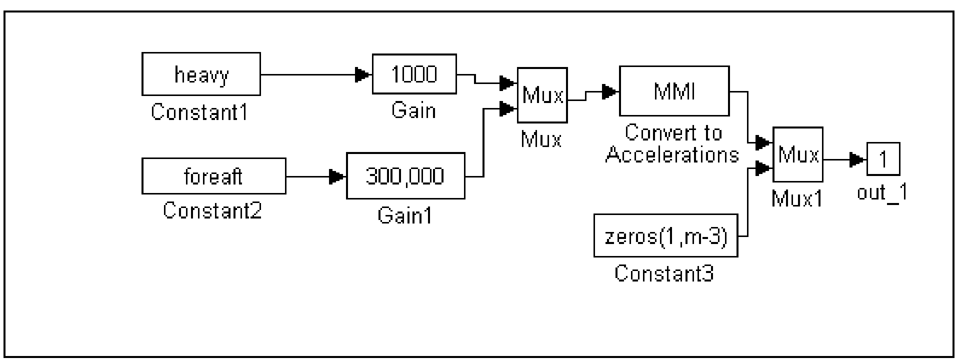

Figure 12. SIMULINK® trim model

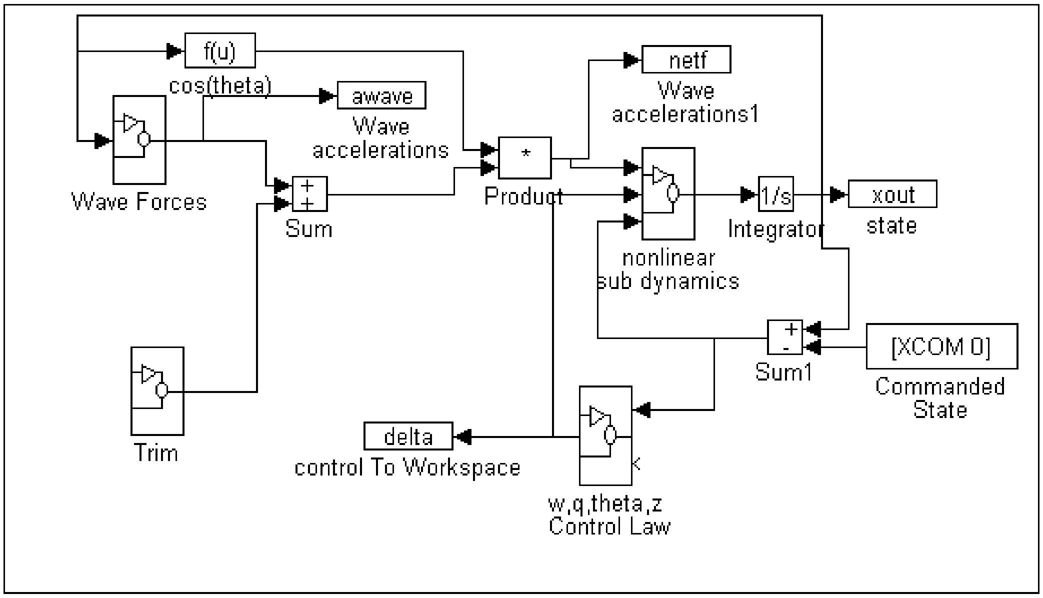

Figure 13. SIMULINK® state feedback control submarine model

Integral control on depth

To apply integral control, an additional state is introduced. Equations (74) through (77) are augmented by:

$$\dot{z}_I = z - z_{commanded} \tag{79}$$

which is used to provide state feedback. This forces the steady state value of z to zero. In general, this approach is satisfactory as long as the control effort does not become saturated and the eigenvalues of the integral state are slower than the state which is being zeroed.

Feedforward of wave forces

Given the wave forces values, control effort can be directly applied to eliminate the average depth error. With a constant disturbance, a steady state value of the depth error can be determined (Appendix B). Using the linear equations of motion, the steady state depth error can be written as a linear combination of the net force and moment disturbances:

$$z_{ss} - z_{commanded} = C_1 F_d + C_2 M_d \tag{80}$$

To eliminate the depth error, it is required to apply the control effort it applied:

$$K_5 = z_{ss}\begin{bmatrix} K_{14} \\ K_{24} \end{bmatrix} \tag{81}$$

Equations (80) and (81) can be combined to give a matrix gain relationship between the net disturbance and the feedforward:

$$K_5 = \begin{bmatrix} K_{14} \\ K_{24} \end{bmatrix}\begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} F_d \\ M_d \end{bmatrix} = \begin{bmatrix} C_1 K_{14} & C_2 K_{14} \\ C_1 K_{24} & C_2 K_{24} \end{bmatrix}\begin{bmatrix} F_d \\ M_d \end{bmatrix} \tag{82}$$

The state feedback control law with feedforward is:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = Kx + K_5 \tag{83}$$

It has been suggested (Musker, Loader, and Butcher, 1988), (Ni, Zhang, and Dai, 1994) that effective periscope depth control can be achieved by feeding forward the average second order wave forces. Because wave forces are a dynamic disturbance and the feedforward was calculated for a steady disturbance, a filter was employed to cut out the high frequency components of the wave forces. The filter employed was a first order Butterworth filter with a cut off frequency ω_{co}. The cut off frequency was initially chosen as one radian/second. This was well below the maximum frequency wave force components (around 2.2 radians/second). Figure 14 and Figure 15 show the effects of the first order Butterworth filter on the wave forces at a depth of 55 feet in sea state three. It is apparent that with a cutoff frequency of ten radians per second, the filtered forces and moments are very close to the unfiltered. At the lower cutoff frequency, 0.1 radians per second, the filtered forces and moments are much closer to the average values.

To implement the feedforward control law, it was assumed that the net external force and moment were known quantities. Equation (82) was implemented in the SIMULINK® model shown in Figure 16 while the complete system model is shown in Figure 17.

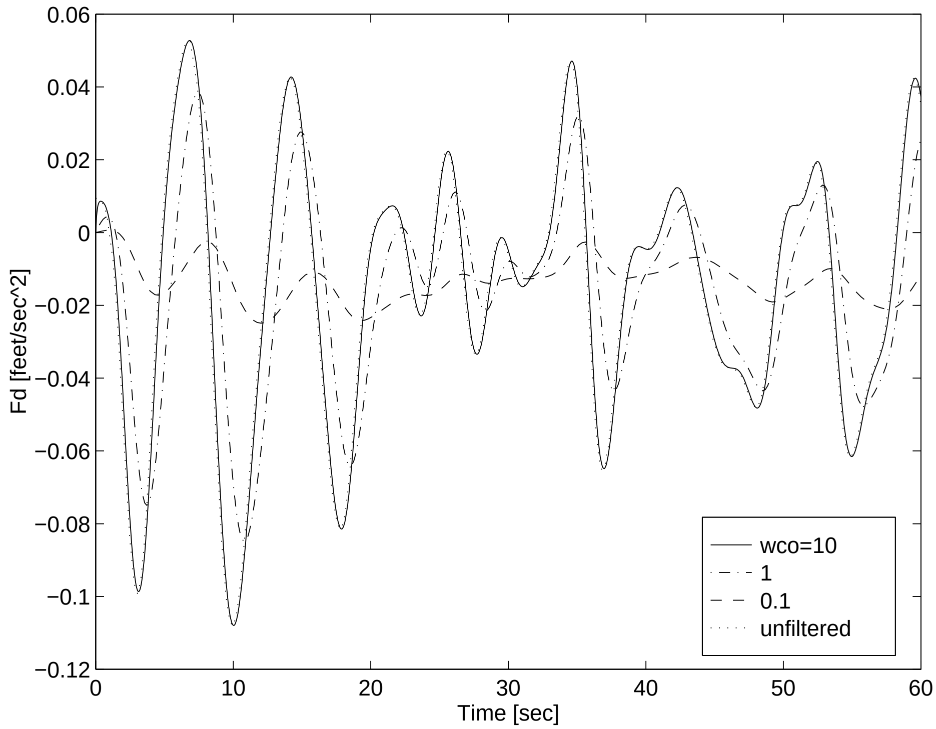

Figure 14. Filtered wave forces for sea state three (head seas)

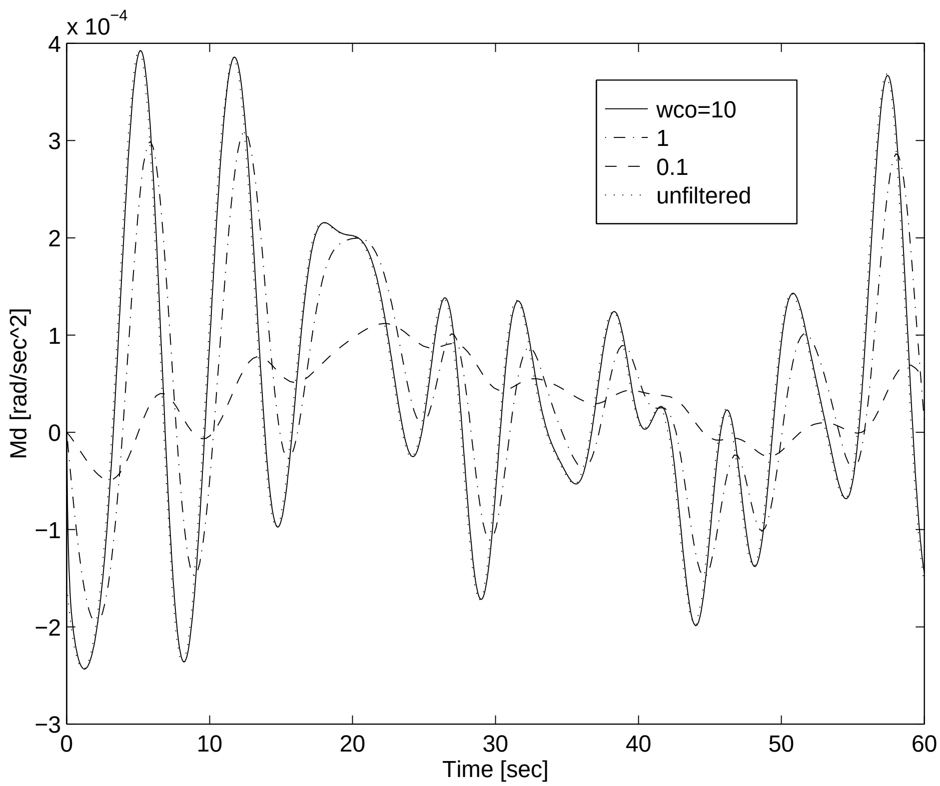

Figure 15. Filtered wave moments for sea state three (head seas)

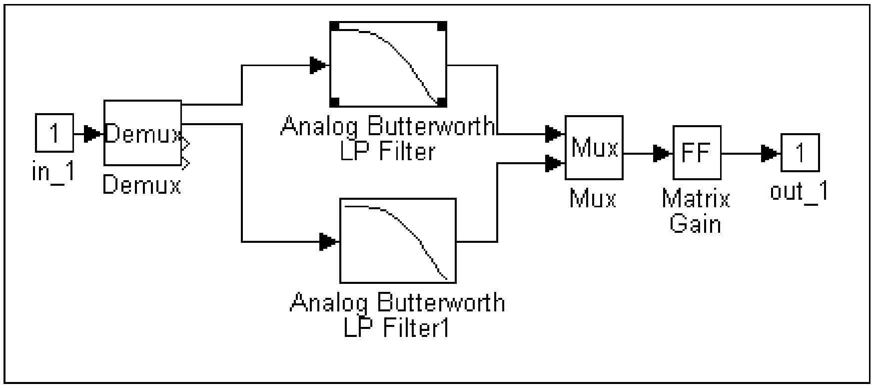

Figure 16. SIMULINK® model of feedforward calculator

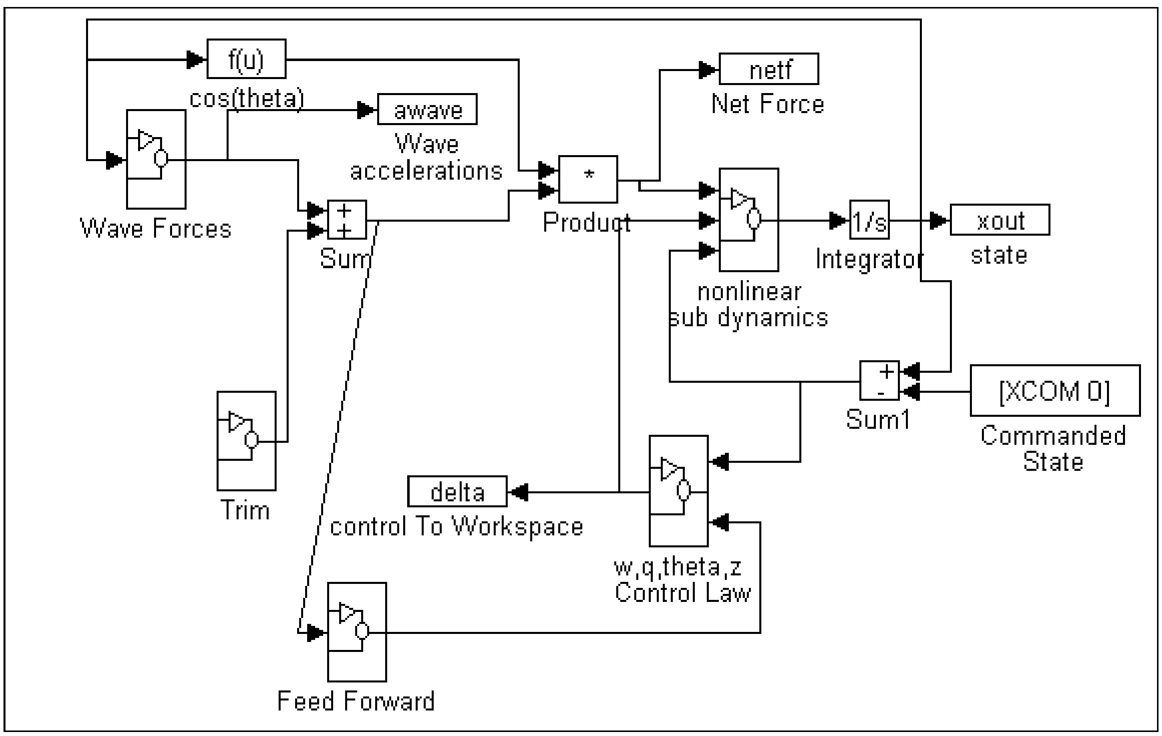

Figure 17. SIMULINK® model of system with feedforward term

Optimization algorithm and parameters

One difficulty of using partial state feedback is that conventional pole placement or linear quadratic regulator algorithms can not be used to determine the gains. The gains in question were selected randomly, and gain combinations which gave stable eigenvalues were simulated. Because of the clamping on the maximum planes angle, some gains which yielded stable eigenvalues resulted in unstable ship control.

Randomly selected gains certainly provide less than optimum depthkeeping. Because of this, each feedback case was optimized to provide the best case for a particular sea state and commanded depth combination. In conjunction with the feedback optimization, the trim was optimized.

The MATLAB® function CONSTR was used to perform the optimizations. CONSTR uses the Broyden-Fletcher-Goldfarb-Shanno variable metric method, and supports constrained optimization problems. To prevent the optimizer from selecting unstable systems, a constraint was placed on the eigenvalues. The real part of the eigenvalues was required to be less than a maximum value, usually -10^{-3}.

The objective of the optimizations was to reduce the root mean square (RMS) value of the depth error. For the basic state feedback control, the average depth was expected to differ somewhat from the commanded depth of 55 feet. Because of this, the objective for these optimizations was to minimize the RMS value of the difference between the depth and the mean depth.

Because the optimizations were performed without regard for minimizing control effort and or rates, large gains with attendant control chatter was expected. Although control chatter is not consistent with normal submarine operations, it was neglected to provide a clear basis of comparison between the differing levels of feedback.

FEEDBACK OF DEPTH AND PITCH ANGLE

Basic control

An elementary level of ship control can be conducted with the stern and bow planesman, each operating to control one particular state. The logical approach to this is for the stern planesman to control the ship's angle, and the bow planesman to control depth. This results in the following control law:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & K_{14} \\ 0 & 0 & K_{23} & 0 \end{bmatrix} \begin{bmatrix} w - w_{commanded} \\ q - q_{commanded} \\ \theta - \theta_{commanded} \\ z - z_{commanded} \end{bmatrix} \tag{84}$$

$$|\delta| \le \delta_{\max} \tag{85}$$

After a stable set of random gains was determined, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement (Vanderplaats, 1984, p. 9) is:

Minimize:

$$F(K_{14}, K_{23}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{mean})^2 \, dt}{t_f}} \tag{86}$$

where:

$z = depth$, determined by nonlinear simulation

$$z_{mean} = \frac{\int_0^{t_f} (z) dt}{t_f},$$

$H =$ *Ballast added to center of buoyancy, thousands of pounds*

$F =$ *Ballast shifted from forward to aft, thousands of pounds*

Subject to:

$$real(eigenvalues(A_c)) \le E_{\max} \tag{87}$$

Deviation from the mean value of depth, vice the commanded was used because of the expected average depth error.

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimized response is shown in Figure 18. The results of the four optimizations are shown in Table 3. For the RMS error and maximum error, the optimized values are given, along with their percentage of the initial values.

In all cases, use of the optimization resulted in reduction of the mean square depth error (measured from the average depth). Reduction of the maximum error was also achieved.

Sea State/Direction	3/head	3/beam	4/head	4/beam
Initial Values				
K_{14}	0.1465	0.1465	0.1465	0.1465
K_{23}	17.51	17.51	17.51	17.51
H/F (10^3 pounds)	15/0	15/0	15/0	15/0
Mean Depth (feet)	55.15	55.20	55.09	55.29
RMS Error (feet)	0.9220	0.9210	1.23	1.30
Maximum Error (feet)	2.46	2.47	3.86	4.76
Eigenvalues	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i
Optimized Values				
K_{14}	0.567	0.2293	0.4708	0.2016
K_{23}	63.83	22.5724	48.6186	19.58
H/F (10^3 pounds)	9.9 / 2.0	15.8/-4.9	15.6/-1.5	4.9/-5
Mean Depth (feet)	54.7	55.99	55.12	55.44
RMS Error (feet)	0.4550 (49%)	0.7549 (82%)	0.657 (53%)	1.23 (95%)
Maximum Error (feet)	1.533 (62%)	2.03 (82%)	2.54 (66%)	4.15 (87%)
Eigenvalues	-0.0388 + 0.2392i -0.0388 - 0.2392i -0.0042 + 0.3911i -0.0042 - 0.3911i	-0.0419 + 0.1464i -0.0419 - 0.1464i -0.0010 + 0.2346i -0.0010 - 0.2346i	-0.0419 + 0.2178i -0.0419 - 0.2178i -0.0010 + 0.3397i -0.0010 - 0.3397i	-0.0010 + 0.2194i -0.0010 - 0.2194i -0.0419 + 0.1358i -0.0419 - 0.1358i

Table 3. Optimized pitch and depth control law results and performance

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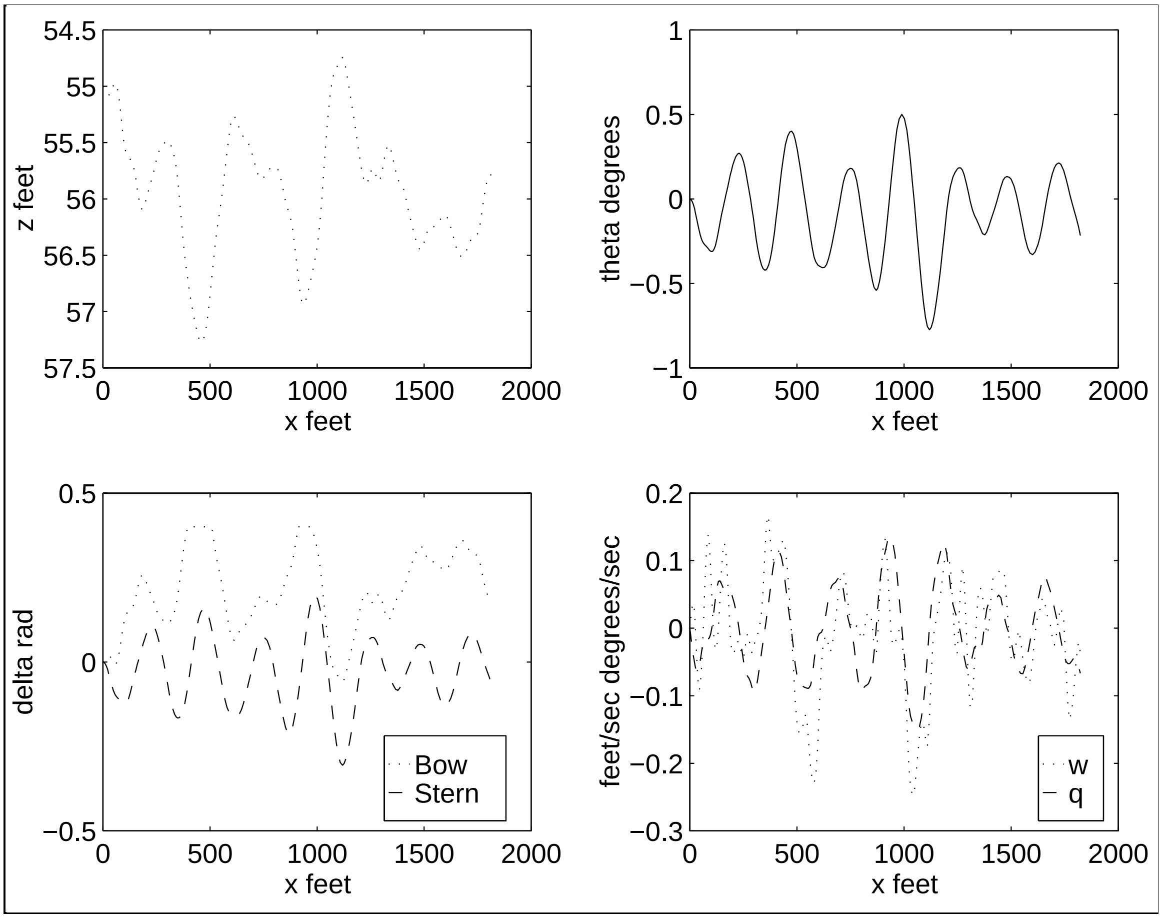

Figure 18. Simulation with depth and pitch angle control in sea state three (head sea direction)

Disturbance feedforward

The pitch angle and depth feedback control can be implemented with a disturbance feedforward to correct average depth error. This results in the following control law:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & K_{14} \\ 0 & 0 & K_{23} & 0 \end{bmatrix} \begin{bmatrix} w - w_{commanded} \\ q - q_{commanded} \\ \theta - \theta_{commanded} \\ z - z_{commanded} \end{bmatrix} + \begin{bmatrix} C_1 K_{14} & C_2 K_{14} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{F}_d \\ \hat{M}_d \end{bmatrix} \tag{88}$$

$$|\delta| \le \delta_{\max} \tag{89}$$

where K_5 is given by Equation (82) and the force and moment disturbances are filtered.

After a stable set of random gains was determined, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement is:

$$F(K_{14}, K_{23}, \omega_{co}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{commanded})^2 \, dt}{t_f}} \tag{90}$$

Subject to:

$$real(eigenvalues(A_c)) \leq E_{\max} \tag{91}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimized response is shown in Figure 19. The results of the four optimizations are shown in Table 4. For the RMS error and maximum error, the optimized values are given, along with their percentage of the initial values.

Sea State/Direction	3/head	3/beam	4/head	4/beam
Initial Values				
K_{14}	0.1465	0.1465	0.1465	0.1465
K_{23}	17.51	17.51	17.51	17.51
ω_{co} (rad/sec)	1	1	1	1
H/F (10^3 pounds)	20/0	20/0	20/0	20/0
Mean Depth (feet)	55.07	56.7	55.826	61.33
RMS Error (feet)	0.408	2.24	1.71	6.93
Maximum Error (feet)	1.104	5.27	3.71	13.46
Eigenvalues	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i	-0.0074 + 0.2096i -0.0074 - 0.2096i -0.0356 + 0.1144i -0.0356 - 0.1144i
Optimized Values				
K_{14}	1.116	3.5763	7.40	3.396
K_{23}	151.00	454.7	1,073.6	979.02
ω_{co} (rad/sec)	0.743	3.30	6.83	6.43
H/F (10^3 pounds)	19.5/3.5	22.1/1.3	26.5/3.25	8.4/-4.0
Mean Depth (feet)	54.996	55.14	55.04	55.21
RMS Error (feet)	0.102 (25%)	0.556 (25%)	0.810 (47%)	0.883 (13%)
Maximum Error (feet)	0.322 (29%)	2.24 (43%)	0.560 (15%)	3.36 (25%)
Eigenvalues	-0.0337 + 0.3345i -0.0337 - 0.3345i -0.0092 + 0.6088i -0.0092 - 0.6088i	-0.0354 + 0.6055i -0.0354 - 0.6055i -0.0076 + 1.0490i -0.0076 - 1.0490i	-0.0322 + 0.8604i -0.0322 - 0.8604i -0.0107 + 1.6296i -0.0107 - 1.6296i	-0.0250 + 0.5675i -0.0250 - 0.5675i -0.0179 + 1.6019i -0.0179 - 1.6019i

Table 4. Optimized pitch and depth control law with disturbance feedforward results and performance

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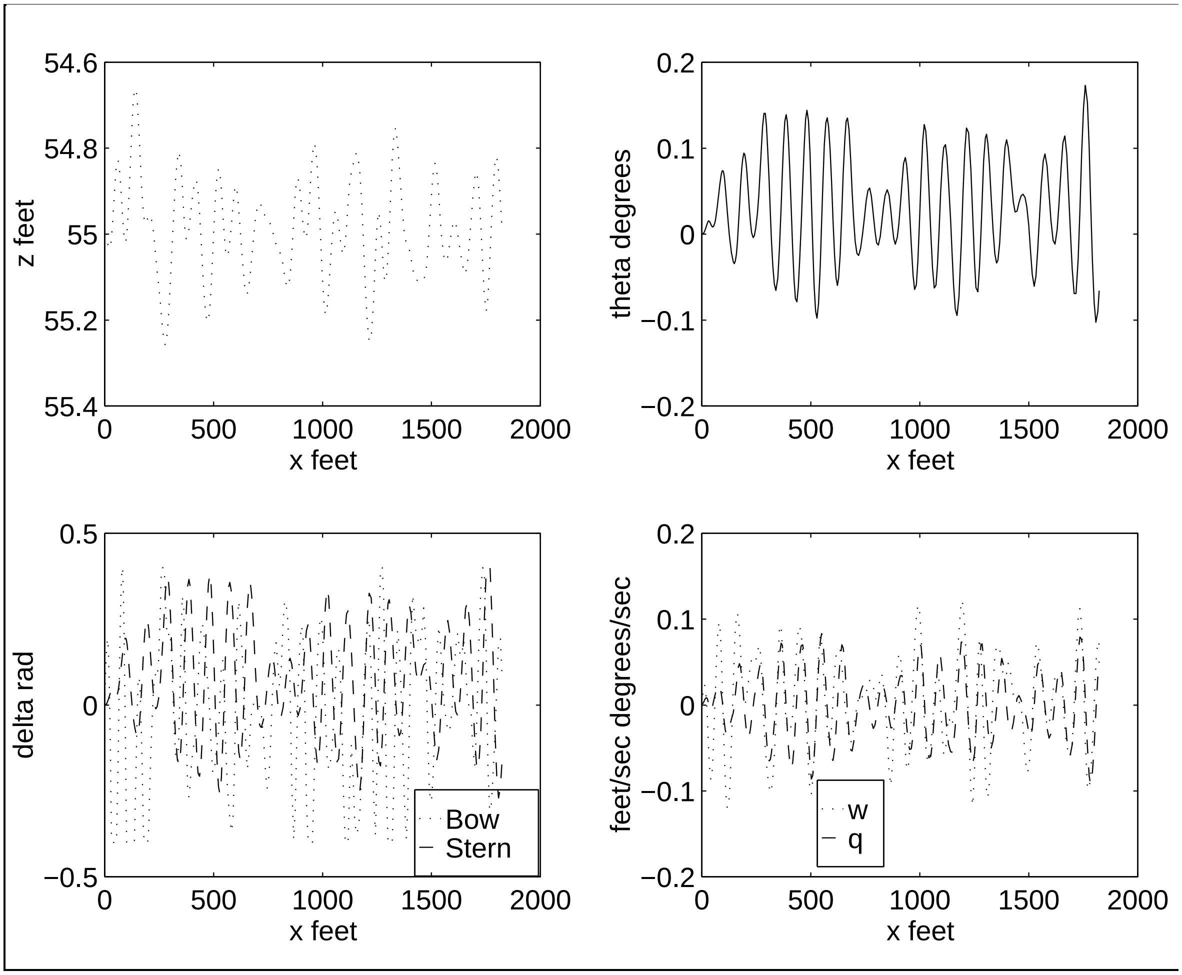

Figure 19. Simulation with depth and pitch angle control with disturbance feedforward, sea state three (head seas)

Integral control

The feedback of depth and pitch angle can be augmented with integral control on depth to remove the average depth error. Since the bow planes are principally used for the control of depth, the integral control was applied to the bow planes only. This results in the following control law:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & K_{14} & K_{15} \\ 0 & 0 & K_{23} & 0 & 0 \end{bmatrix} \begin{bmatrix} w - w_{commanded} \\ q - q_{commanded} \\ \theta - \theta_{commanded} \\ z - z_{commanded} \\ z_I \end{bmatrix} \tag{92}$$

$$|\delta| \leq \delta_{\max} \tag{93}$$

After a stable set of random gains was determined, the controller was optimized to minimize the deviation from the commanded depth. The formal optimization statement is:

Minimize:

$$F(K_{14}, K_{15}, K_{23}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{commanded})^2 \, dt}{t_f}} \tag{94}$$

Subject to:

$$real(eigenvalues(A_c)) \leq E_{\max} \tag{95}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimized response is shown in Figure 20. The results of the four optimizations are shown in Table 5.

Sea State/Direction	3/head	3/beam	4/head	4/beam
Initial Values				
K_{14}	0.1465	0.1465	0.1465	0.1465
K_{15}	0.001	0.001	0.001	0.001
K_{23}	17.51	17.51	17.51	17.51
H/F (10^3 pounds)	20/0	20/0	20/0	20/0
Mean Depth (feet)	55.15	55.16	55.19	55.24
RMS Error (feet)	0.987	0.986	1.29	1.39
Maximum Error (feet)	2.614	2.63	4.01	5.12
Eigenvalues	-0.0077 + 0.2088i -0.0077 - 0.2088i -0.0318 + 0.1148i -0.0318 - 0.1148i -0.0070	-0.0077 + 0.2088i -0.0077 - 0.2088i -0.0318 + 0.1148i -0.0318 - 0.1148i -0.0070	-0.0077 + 0.2088i -0.0077 - 0.2088i -0.0318 + 0.1148i -0.0318 - 0.1148i -0.0070	-0.0077 + 0.2088i -0.0077 - 0.2088i -0.0318 + 0.1148i -0.0318 - 0.1148i -0.0070
Optimized Values				
K_{14}	1.5609	0.6329	0.2906	0.296
K_{15}	0.0019	0.0012	0.0004	0.0005
K_{23}	304.5	107.76	28.46	76.53
H/F (10^3 pounds)	18.2/1.4	20.0/0.1	18.1/-3.4	12.2/-4.2
Mean Depth (feet)	55.01	55.09	55.05	55.11
RMS Error (feet)	0.455 (46%)	0.3811 (39%)	0.865 (67%)	1.05 (76%)
Maximum Error (feet)	2.035 (78%)	1.0138 (39%)	3.38 (84%)	3.53 (69%)
Eigenvalues	-0.0149 + 0.8824i -0.0149 - 0.8824i -0.0274 + 0.3887i -0.0274 - 0.3887i -0.0012	-0.0134 + 0.5221i -0.0134 - 0.5221i -0.0286 + 0.2475i -0.0286 - 0.2475i -0.0020	-0.0001 + 0.2615i -0.0001- 0.2615i -0.0421 + 0.1675i -0.0421 - 01675i -0.0015	-0.0163 + 0.4296i -0.0163 - 0.4296i -0.0258 + 0.1707i -0.0258 - 0.1707i -0.0015

Table 5. Optimized pitch and depth integral control law results and performance

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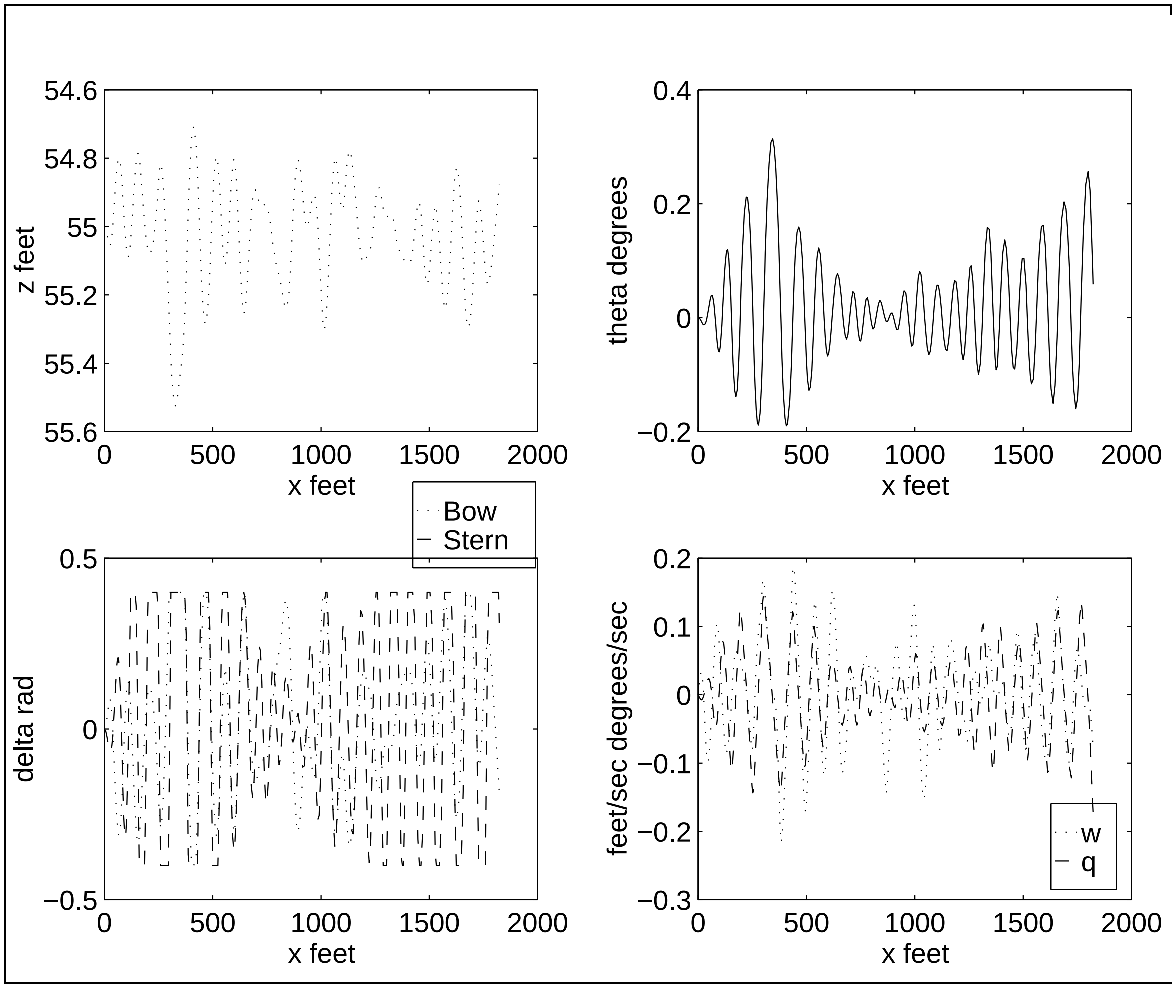

Figure 20. Simulation with depth, pitch angle, and integral control, sea state three (head seas)

FULL STATE FEEDBACK WITH PARTIAL DISTRIBUTION

Basic control

The poor depth control of the previous section can be improved be adding to the number of states fed back. In keeping with previous logic, the bow planes will be controlled by the depth and heave, while the stern planes will be controlled by pitch and pitch rate. This results in the following control law:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = \begin{bmatrix} K_{11} & 0 & 0 & K_{14} \\ 0 & K_{22} & K_{23} & 0 \end{bmatrix} \begin{bmatrix} w - w_{commanded} \\ q - q_{commanded} \\ \theta - \theta_{commanded} \\ z - z_{commanded} \end{bmatrix} \tag{96}$$

$$|\delta| \le \delta_{\max} \tag{97}$$

After a stable set of random gains was determined, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement is:

Minimize:

$$F(K_{11}, K_{14}, K_{22}, K_{23}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{mean})^2 \, dt}{t_f}} \tag{98}$$

Subject to:

$$real(eigenvalues(A_c)) \le E_{\max} \tag{99}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimized response is shown in Figure 21. The results of the four optimizations are shown in Table 6.

Sea State/Direction	3/head		3/beam		4/head		4/beam	
Initial Values								
K^T	12.5543	0	12.5543	0	12.5543	0	12.5543	0
	0	22.5268	0	22.5268	0	22.5268	0	22.5268
	0	24.9900	0	24.9900	0	24.9900	0	24.9900
	2.5490	0	2.5490	0	2.5490	0	2.5490	0
H/F (103 pounds)	20/0		20/0		20/0		20/0	
Mean Depth (feet)	55.06		55.33		55.21		55.73	
RMS Error (feet)	0.216		0.425		0.412		1.06	
Maximum Error (feet)	0.796		1.38		1.26		3.38	
Eigenvalues	-0.6743 -0.0413 + 0.3587i -0.0413 - 0.3587i -0.1789		-0.6743 -0.0413 + 0.3587i -0.0413 - 0.3587i -0.1789		-0.6743 -0.0413 + 0.3587i -0.0413 - 0.3587i -0.1789		-0.6743 -0.0413 + 0.3587i -0.0413 - 0.3587i -0.1789	
Optimized Values								
K^T		0	7.035	0	13.265	0	16.58	0
	0	1366.3	0	91.224	0	91.49	0	96.44
	0	1163.3	0	20.87	0	51.15	0	82.43
	89.26	0	3.813	0	2.1048	0	15.33	0
H/F (103 pounds)	14.6/-1.4		12.3/-0.4		20.0/0.1		16.8/0.0	
Mean Depth (feet)	55.02		55.01		55.21		55.09	
RMS Error (feet)	0.1969 (91%)		0.350 (82%)		0.358 (87%)		0.821 (77%)	
Maximum Error (feet)	1.17 (147%)		0.99 (72%)		1.13 (90%)		3.46 (102%)	
Eigenvalues	-3.3197 -0.0693 + 1.2522i -0.0693 - 1.2522i -0.1879		-0.1266 + 0.4709i -0.1266 - 0.4709i -0.2619 + 0.0179i -0.2619 - 0.0179i		-0.7286 -0.1458 + 0.4700i -0.1458 - 0.4700i -0.1514		-0.2213 + 0.8861i -0.2213 - 0.8861i -0.3797 + 0.4819i -0.3797 - 0.4819i	

Table 6. Full state feedback (partial distribution) control law optimization results and performance

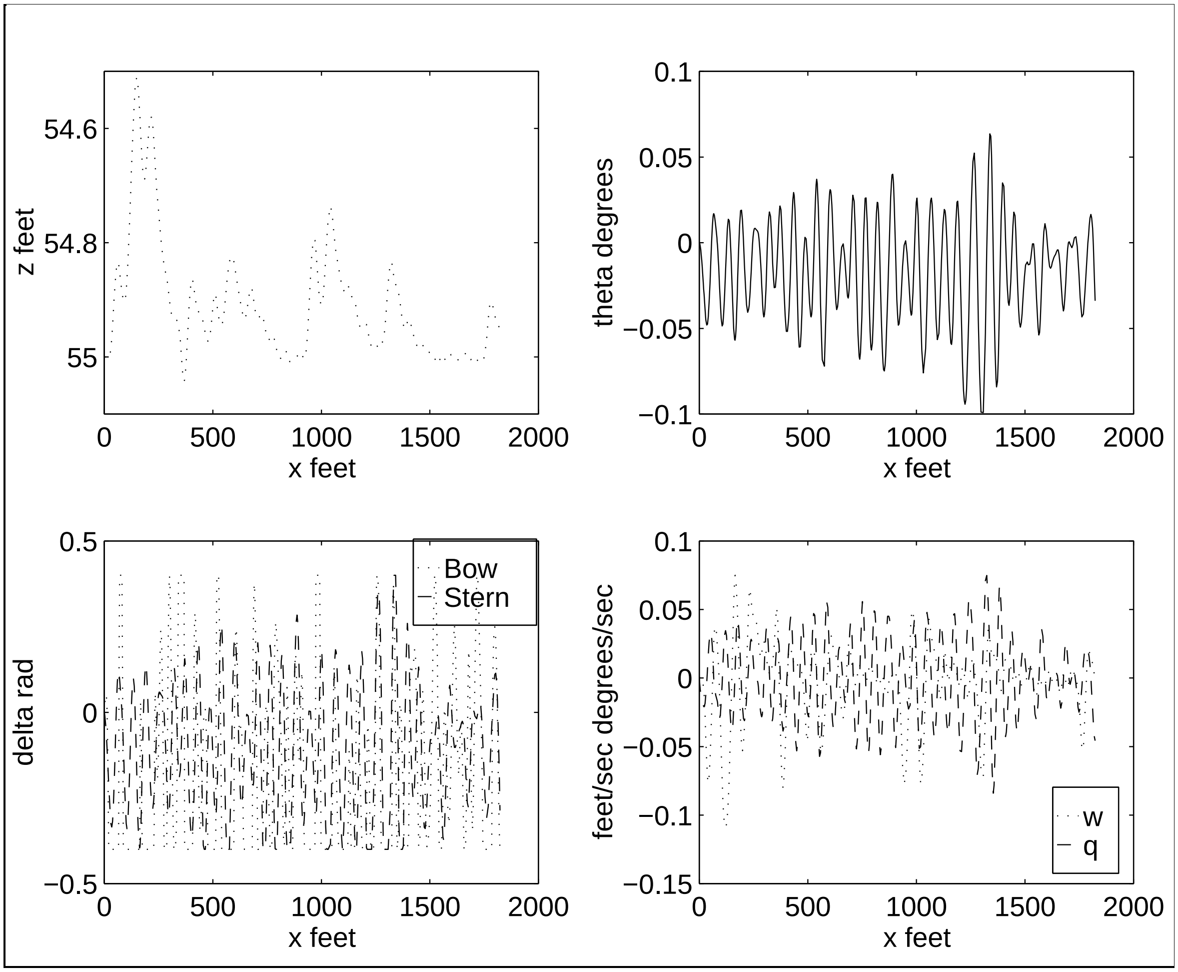

Figure 21. Simulation with full state partial distribution feedback control, sea state three (head seas)

Disturbance feedforward

As before, the basic control law can be modified to include a feedforward term to correct the average depth error.

$$\begin{bmatrix}\delta_{bp}\\ \delta_{sp}\end{bmatrix}=\begin{bmatrix}K_{11} & 0 & 0 & K_{14}\\ 0 & K_{22} & K_{23} & 0\end{bmatrix}\begin{bmatrix}w-w_{commanded}\\ q-q_{commanded}\\ \theta-\theta_{commanded}\\ z-z_{commanded}\end{bmatrix}+\begin{bmatrix}C_1K_{14} & C_2K_{14}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\hat{F}_d\\ \hat{M}_d\end{bmatrix} \tag{100}$$

$$|\delta|\leq\delta_{\max} \tag{101}$$

After a stable set of random gains was determined, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement is:

Minimize:

$$F(K_{11}, K_{14}, K_{22}, K_{23}, \omega_{co}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{commanded})^2 \, dt}{t_f}} \tag{102}$$

Subject to:

$$real(eigenvalues(A_c)) \leq E_{\max} \tag{103}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimized response is shown in Figure 22. The results of the four optimizations are shown in Table 7.

<table>
<tr><th>Sea State/Direction</th><th colspan="2">3/head</th><th colspan="2">3/beam</th><th colspan="2">4/head</th><th colspan="2">4/beam</th></tr>
<tr><td>Initial Values</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>K^T</td><td>12.5543
0
0
2.5490</td><td>0
22.5268
24.9900
0</td><td>12.5543
0
0
2.5490</td><td>0
22.5268
24.9900
0</td><td>12.5543
0
0
2.5490</td><td>0
22.5268
24.9900
0</td><td>12.5543
0
0
2.5490</td><td>0
22.5268
24.9900
0</td></tr>
<tr><td>ω_{co} (rad/sec)</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td></tr>
<tr><td>H/F (103 pounds)</td><td colspan="2">20/0</td><td colspan="2">20/0</td><td colspan="2">20/0</td><td colspan="2">20/0</td></tr>
<tr><td>Mean Depth (feet)</td><td colspan="2">55.16</td><td colspan="2">55.35</td><td colspan="2">55.23</td><td colspan="2">55.82</td></tr>
<tr><td>RMS Error (feet)</td><td colspan="2">0.371</td><td colspan="2">0.533</td><td colspan="2">0.551</td><td colspan="2">1.40</td></tr>
<tr><td>Maximum Error (feet)</td><td colspan="2">1.264</td><td colspan="2">1.83</td><td colspan="2">1.72</td><td colspan="2">3.50</td></tr>
<tr><td>Eigenvalues</td><td colspan="2">-0.6743
-0.0413 + 0.3587i
-0.0413 - 0.3587i
-0.1789</td><td colspan="2">-0.6743
-0.0413 + 0.3587i
-0.0413 - 0.3587i
-0.1789</td><td colspan="2">-0.6743
-0.0413 + 0.3587i
-0.0413 - 0.3587i
-0.1789</td><td colspan="2">-0.6743
-0.0413 + 0.3587i
-0.0413 - 0.3587i
-0.1789</td></tr>
<tr><td>Optimized Values</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>K^T</td><td>25.81
0
0
3.5847</td><td>0
175.77
26.1
0</td><td>15.73
0
0
9.446</td><td>0
230.65
535.98
0</td><td>12.74
0
0
2.70</td><td>0
18.09
20.1
0</td><td>16.87
0
0
3.714</td><td>0
9.197
18.12
0</td></tr>
<tr><td>ω_{co} (rad/sec)</td><td colspan="2">0.481</td><td colspan="2">3.80</td><td colspan="2">0.998</td><td colspan="2">0.999</td></tr>
<tr><td>H/F (10^3 pounds)</td><td colspan="2">7.1/-3.3</td><td colspan="2">19.7/-2.5</td><td colspan="2">20.0/0.0</td><td colspan="2">19.7/0.0</td></tr>
<tr><td>Mean Depth (feet)</td><td colspan="2">55.01</td><td colspan="2">55.23</td><td colspan="2">55.18</td><td colspan="2">55.63</td></tr>
<tr><td>RMS Error (feet)</td><td colspan="2">0.0785 (21%)</td><td colspan="2">0.3624 (68%)</td><td colspan="2">0.5171 (94%)</td><td colspan="2">1.25 (89%)</td></tr>
<tr><td>Maximum Error (feet)</td><td colspan="2">0.246 (19%)</td><td colspan="2">1.07 (58%)</td><td colspan="2">2.03 (118%)</td><td colspan="2">3.26 (93%)</td></tr>
<tr><td>Eigenvalues</td><td colspan="2">-1.1709
-0.5214
-0.0948
-0.3993</td><td colspan="2">-0.1692 + 1.3601i
-0.1692 - 1.3601i
-0.6826 + 0.4529i
-0.6826 - 0.4529i</td><td colspan="2">-0.6730
-0.0429 + 0.3320i
-0.0429 - 0.3320i
-0.1767</td><td colspan="2">-0.9159
-0.0386 + 0.3173i
-0.0386 - 0.3173i
-0.1769</td></tr>
</table>

Table 7. Full state feedback (partial distribution) with disturbance feedforward control law optimization results and performance

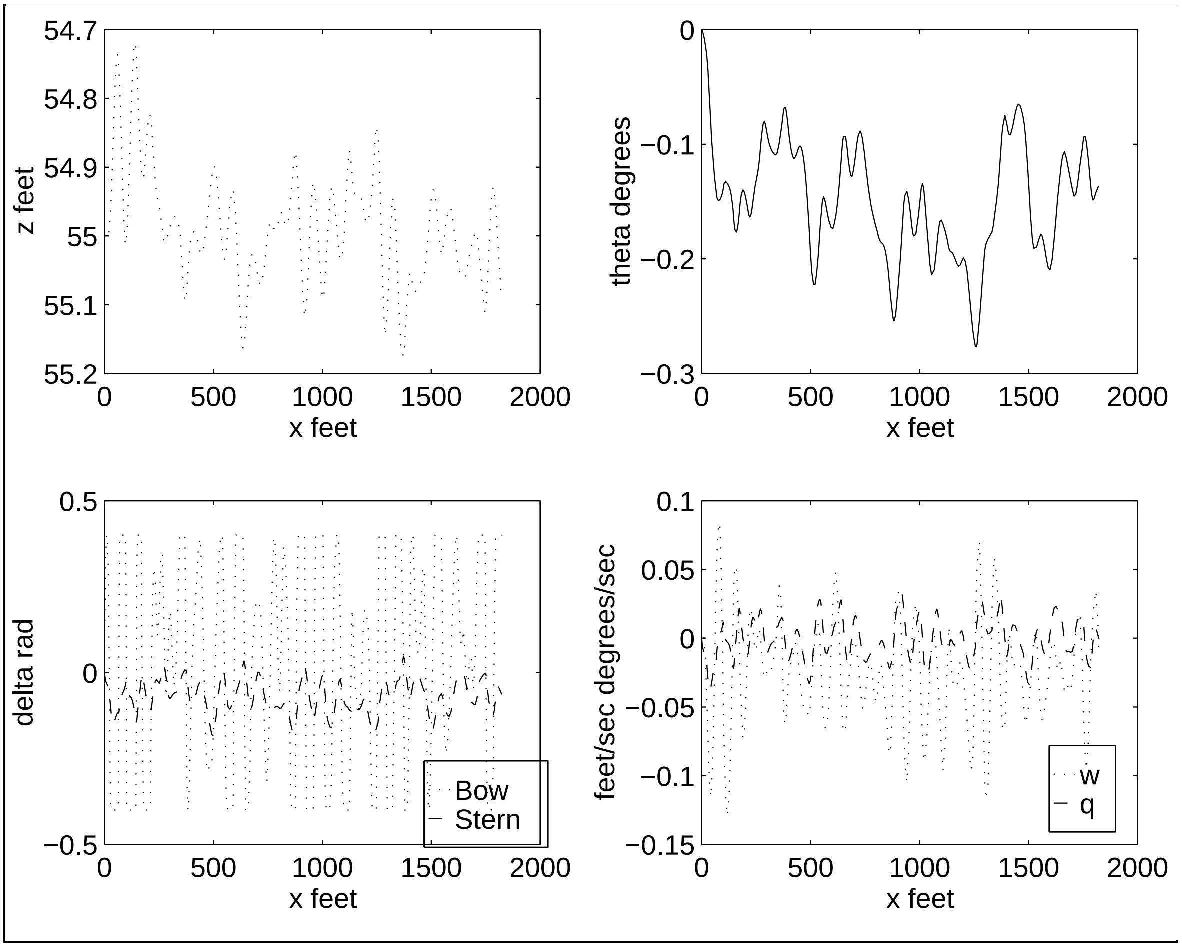

Figure 22. Simulation with full state partial distribution control and disturbance feedforward, sea state three (head seas)

Integral Control

This full state feedback with partial distribution was augmented with integral control on depth to remove the average depth error. As before, the integral control was done using the bow planes only. This results in the following control law:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = \begin{bmatrix} K_{11} & 0 & 0 & K_{14} & K_{15} \\ 0 & K_{22} & K_{23} & 0 & 0 \end{bmatrix} \begin{bmatrix} w - w_{commanded} \\ q - q_{commanded} \\ \theta - \theta_{commanded} \\ z - z_{commanded} \\ z_I \end{bmatrix} \tag{104}$$

$$|\delta| \le \delta_{\max} \tag{105}$$

After a stable set of random gains was determined, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement is as follows:

Minimize:

$$F(K_{11}, K_{14}, K_{15}, K_{22}, K_{23}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{mean})^2 \, dt}{t_f}} \tag{106}$$

Subject to:

$$real(eigenvalues(A_c)) \le E_{\max} \tag{107}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimized response is shown in Figure 23. The results of the four optimizations are shown in Table 8.

Sea State/Direction	3/head		3/beam		4/head		4/beam	
Initial Values								
K^T	12.5543 0 0 2.5490 0.0010	0 22.5268 24.9900 0 0	12.5543 0 0 2.5490 0.0010	0 22.5268 24.9900 0 0	12.5543 0 0 2.5490 0.0010	0 22.526 8 24.990 0	12.5543 0 0 2.5490 0.0010	0 22.526 8 24.99 0
H/F (10^3 pounds)	20/0		20/0		20/0		20/0	
Mean Depth (feet)	55.05		55.31		55.21		55.74	
RMS Error (feet)	0.2106		0.533		0.4868		1.205	
Maximum Error (feet)	0.866		1.723		1.82		3.43	
Eigenvalues	-0.6744 -0.0412 + 0.3587i -0.0412 - 0.3587i -0.1785 -0.0004		-0.6744 -0.0412 + 0.3587i -0.0412 - 0.3587i -0.1785 -0.0004		-0.6744 -0.0412 + 0.3587i -0.0412 - 0.3587i -0.1785 -0.0004		-0.6744 -0.0412 + 0.3587i -0.0412 - 0.3587i -0.1785 -0.0004	
Optimized Values								
K^T	523.12 0 0 89.26 .04016	0 1366.3 1163.3 0 0	385.2 0 0 254.4 0.0077	0 874.3 1400.5 0 0	14.074 0 0 2.146 0.0003	0 73.89 21.85 0 0	79.08 0 0 52.88 0.0032	0 353.83 54.45 0
H/F (10^3 pounds)	14.6/-1.4		26.7/4.6		21.2/-0.2		10.5/-2.6	
Mean Depth (feet)	55.02		55.05		55.25		55.18	
RMS Error (feet)	0.059 (28%)		0.3017 (57%)		0.4274 (88%)		0.909 (75%)	
Maximum Error (feet)	0.248 (29%)		0.862 (50%)		1.16 (64%)		3.59 (105%)	
Eigenvalues	-30.7579 -4.6840 -1.0562 -0.1695 -0.0005		-22.3751 -1.8146 + 1.6555i -1.8146 - 1.6555i -0.6572 0.00003		-0.7495 -0.1526 + 0.3297i -0.1526 - 0.3297i -0.1168 -0.0001		-3.4631 -1.4563 -0.9504 -0.1490 -0.0001	

Table 8. Full state feedback (partial distribution) integral control law optimization results and performance

.

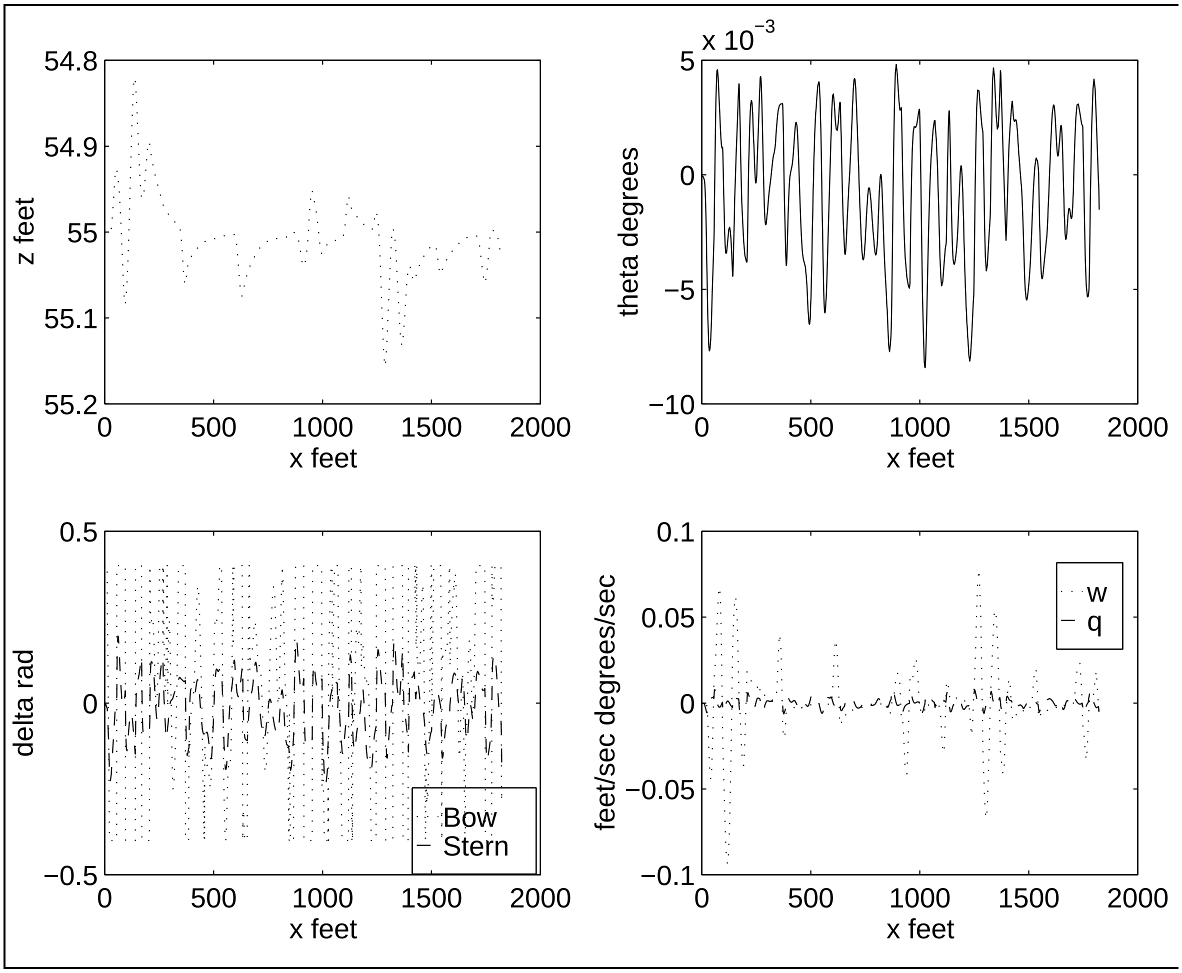

Figure 23. Simulation with full state partial distribution feedback integral control, sea state three (head seas)

FULL STATE FEEDBACK

Basic control

The best control possible using state feedback should result from the use of all four states by each control. This results in the following control law:

$$\begin{bmatrix}\delta_{bp}\\ \delta_{sp}\end{bmatrix}=\begin{bmatrix}K_{11} & K_{12} & K_{13} & K_{14}\\ K_{21} & K_{22} & K_{23} & K_{24}\end{bmatrix}\begin{bmatrix}w-w_{commanded}\\ q-q_{commanded}\\ \theta-\theta_{commanded}\\ z-z_{commanded}\end{bmatrix} \tag{108}$$

$$|\delta| \le \delta_{\max} \tag{109}$$

After a stable set of gains was determined using a linear quadratic regulator algorithm, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement was:

Minimize:

$$F(K_{11},K_{12},K_{13},K_{14},K_{21},K_{22},K_{23},K_{24},H,F)=\sqrt{\frac{\int_0^{t_f}(z-z_{mean})^2\,dt}{t_f}} \tag{110}$$

Subject to:

$$real(eigenvalues(A_c)) \le E_{\max} \tag{111}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimum response is shown in Figure 24. The results of the four optimizations are shown in Table 9.

Sea State/Direction	3/head		3/beam		4/head		4/beam	
Initial Values								
K^T	6.847	5.1622	6.847	5.1622	6.847	5.1622	6.847	5.1622
	-168.26	121.32	-168.26	121.32	-168.26	121.32	-168.26	121.32
	-65.795	27.744	-65.795	27.744	-65.795	27.744	-65.795	27.744
	0.9740	-0.0789	0.9740	-0.0789	0.9740	-0.0789	0.9740	-0.0789
H/F (10^3 pounds)	20/0		20/0		20/0		20/0	
Mean Depth (feet)	55.09		55.37		55.21		56.16	
RMS Error (feet)	0.0914		0.3605		0.355		1.60	
Maximum Error (feet)	0.262		1.34		1.09		4.55	
Eigenvalues	-0.8859		-0.8859		-0.8859		-0.8859	
	-0.2854		-0.2854		-0.2854		-0.2854	
	-0.1630 + 0.2247i		-0.1630 + 0.2247i		-0.1630 + 0.2247i		-0.1630 + 0.2247i	
	-0.1630 - 0.2247i		-0.1630 - 0.2247i		-0.1630 - 0.2247i		-0.1630 - 0.2247i	
Optimized Values								
K^T	10.300	9.638	4.591	11.31	4.591	11.306	3.503	11.5734
	45.790	170.45	-283.9	78.91	-283.9	78.91	-374.02	178.92
	-135.16	39.390	125.7	-1.333	-125.7	-1.333	-66.65	40.760
	3.6389	0.0308	1.457	0.1568	1.457	0.1568	0.446	0.0221
H/F (10^3 pounds)	20.1/-0.9		18.0/-0.3		18.0/-0.33		19.9/0.0	
Mean Depth (feet)	55.03		55.13		55.13		56.54	
RMS Error (feet)	0.037 (40%)		0.2638 (73%)		0.2683 (76%)		1.24 (78%)	
Maximum Error (feet)	0.119 (45%)		0.961 (72%)		0.961 (88%)		4.06 (89%)	
Eigenvalues	-1.7613		-1.0446		-1.0446		-1.0313 + 0.4218i	
	-0.2510		-0.3493 + 0.2881i		-0.3493 + 0.2881i		-1.0313 - 0.4218i	
	-0.0617 + 0.5264i		-0.3493 - 0.2881i		-0.3493 - 0.2881i		-0.0936 + 0.0741i	
	-0.0617 - 0.5264i		-0.2053		-0.2053		-0.0936 - 0.0741i	

Table 9. Full state feedback control law optimization results and performance

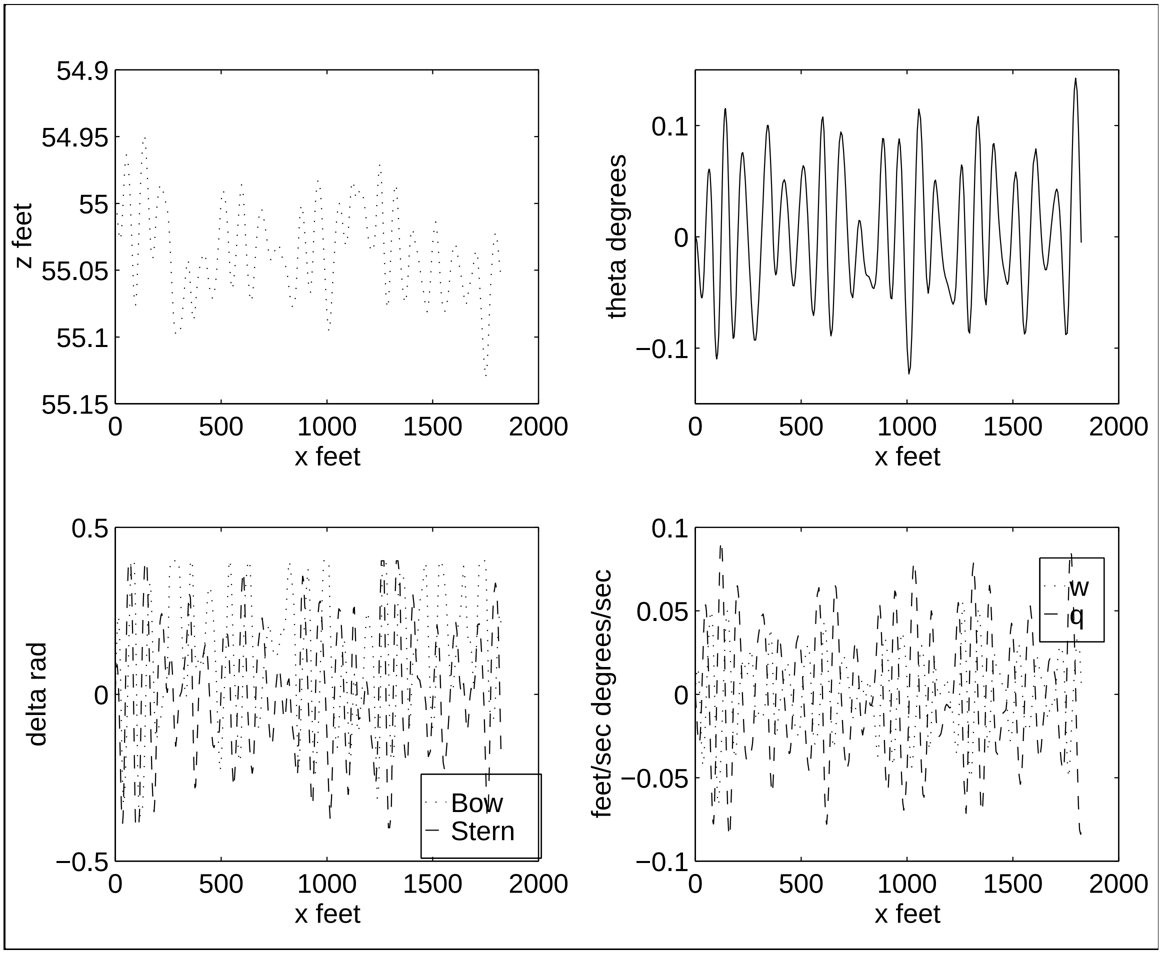

Figure 24. Full state feedback optimized control simulation, sea state three (head seas)

Disturbance feedforward

The state feedback control law was modified to include disturbance feedforward. This results in the following control law:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix} \begin{bmatrix} w - w_{commanded} \\ q - q_{commanded} \\ \theta - \theta_{commanded} \\ z - z_{commanded} \end{bmatrix} + \begin{bmatrix} C_1 K_{14} & C_2 K_{14} \\ C_1 K_{24} & C_2 K_{24} \end{bmatrix} \begin{bmatrix} \hat{F}_d \\ \hat{M}_d \end{bmatrix} \tag{112}$$

$$|\delta| \leq \delta_{\max} \tag{113}$$

After a stable set of gains was determined using a linear quadratic regulator algorithm, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement was:

Minimize:

$$F(K_{11}, K_{12}, K_{13}, K_{14}, K_{21}, K_{22}, K_{23}, K_{24}, \omega_{co}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{commanded})^2 \, dt}{t_f}} \tag{114}$$

Subject to:

$$real(eigenvalues(A_c)) \le E_{\max} \tag{115}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimum response is shown in Figure 25. The results of the four optimizations are shown in Table 10.

Sea State/Direction	3/head		3/beam		4/head		4/beam	
Initial Values								
K^T	6.847	5.1622	6.847	5.1622	6.847	5.1622	6.847	5.1622
	-168.26	121.32	-168.26	121.32	-168.26	121.32	-168.26	121.32
	-65.795	27.744	-65.795	27.744	-65.795	27.744	-65.795	27.744
	0.9740	-0.0789	0.9740	-0.0789	0.9740	-0.0789	0.9740	-0.0789
ω_{co} (rad/sec)	1		1		1		1	
H/F (10^3 pounds)	20/0		20/0		20/0		20/0	
Mean Depth (feet)	55.06		55.937		55.21		56.70	
RMS Error (feet)	0.1428		1.343		0.430		2.393	
Maximum Error (feet)	0.4897		3.46		1.11		5.49	
Eigenvalues	-0.8859		-0.8859		-0.8859		-0.8859	
	-0.2854		-0.2854		-0.2854		-0.2854	
	-0.1630 + 0.2247i		-0.1630 + 0.2247i		-0.1630 + 0.2247i		-0.1630 + 0.2247i	
	-0.1630 - 0.2247i		-0.1630 - 0.2247i		-0.1630 - 0.2247i		-0.1630 - 0.2247i	
Optimized Values								
K^T	-26.2	96.5	0.3964	11.08	20.60	-2.681	5.00	-3.94
	-1633	914.4	-879.14	329.16	-194.6	181.5	-452.53	609.1
	-368.8	91.9	-297.0	-91.096	27.66	48.6	25.47	449.5
	5	-1.2	3.52	0.215	-0.276	-0.238	6.66	0.318
ω_{co} (rad/sec)	1.4046		1.033		0.983		1.739	
H/F (10^3 pounds)	13.5/-0.8		19.16/-0.2134		19.0/-0.1		18.8/-0.1	
Mean Depth (feet)	55.0013		55.18		55.18		55.22	
RMS Error (feet)	0.0928 (65%)		0.4121 (31%)		0.400 (36%)		0.792 (33%)	
Maximum Error (feet)	0.2852 (58%)		1.62 (47%)		1.117 (101%)		2.23 (41%)	
Eigenvalues	-7.0391		-0.8095 + 0.5268i		-1.1927		-0.0898 + 0.9297i	
	-4.6280		-0.8095 - 0.5268i		-0.2499 + 0.3312i		-0.0898 - 0.9297i	
	-0.1322 + 0.1329i		-1.1529		-0.2499 - 0.3312i		-1.1834	
	-0.1322 - 0.1329i		-0.0315		-0.0618		-0.6517	

Table 10. Full state feedback control law with disturbance feedforward optimization results and performance

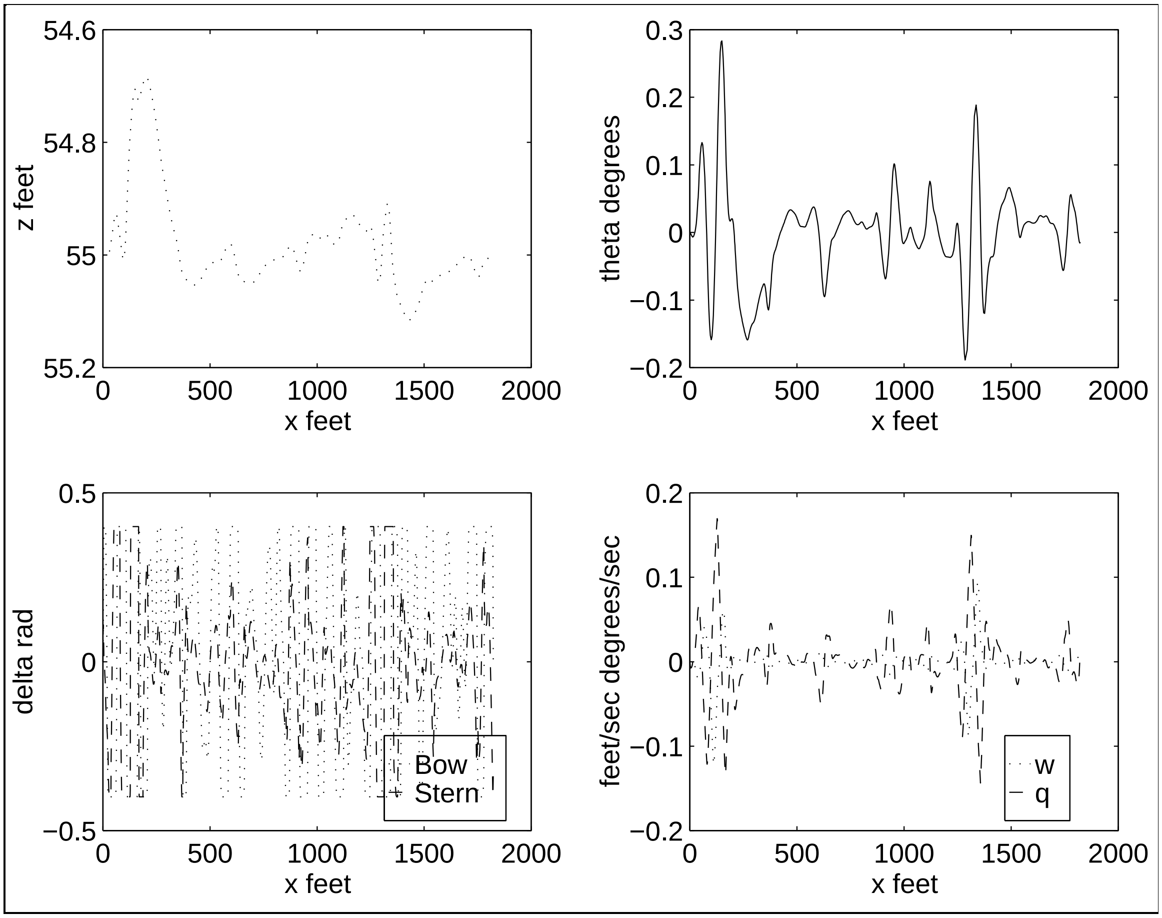

Figure 25. Full state feedback control with disturbance feedforward optimized control simulation, sea state three (head seas)

Integral control

This full state feedback with partial distribution was augmented with integral control on depth to remove the average depth error. Since the bow planes are principally used for the control of depth, the integral control was done using the bow planes only. This results in the following control law:

$$\begin{bmatrix} \delta_{bp} \\ \delta_{sp} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \end{bmatrix} \begin{bmatrix} w - w_{commanded} \\ q - q_{commanded} \\ \theta - \theta_{commanded} \\ z - z_{commanded} \\ z_I \end{bmatrix} \tag{116}$$

$$|\delta| \le \delta_{\max} \tag{117}$$

After a stable set of random gains was determined, the controller was optimized to minimize the deviation from the average depth. The formal optimization statement is:

Minimize:

$$F(K_{11}, K_{12}, K_{13}, K_{14}, K_{15}, K_{21}, K_{22}, K_{23}, K_{24}, K_{25}, H, F) = \sqrt{\frac{\int_0^{t_f} (z - z_{commanded})^2 \, dt}{t_f}} \tag{118}$$

Subject to:

$$real(eigenvalues(A_c)) \leq E_{\max} \tag{119}$$

This approach was used for each of the four sea state cases. For sea state three (head seas), the optimum response is shown in Figure 26. The results of the four optimizations are shown in Table 11.

Sea State/Direction	3/head		3/beam		4/head		4/beam	
Initial Values								
K^T	6.847	5.1622	6.847	5.1622	6.847	5.1622	6.847	5.1622
	-168.26	121.32	-168.26	121.32	-168.26	121.32	-168.26	121.32
	-65.795	27.744	-65.795	27.744	-65.795	27.744	-65.795	27.744
	0.9740	-0.0789	0.9740	-0.0789	0.9740	-0.0789	0.9740	-0.0789
	0.01	-0.01	0.01	-0.01	0.01	-0.01	0.01	-0.01
H/F (10^3 pounds)	20/0		20/0		20/0		20/0	
Mean Depth (feet)	55.01		55.02		54.78		54.68	
RMS Error (feet)	0.101		0.415		0.573		2.483	
Maximum Error (feet)	0.3065		1.545		1.79		6.927	
Eigenvalues	-0.8854		-0.8854		-0.8854		-0.8854	
	-0.2693		-0.2693		-0.2693		-0.2693	
	-0.1652 + 0.2153i		-0.1652 + 0.2153i		-0.1652 + 0.2153i		-0.1652 + 0.2153i	
	-0.1652 - 0.2153i		-0.1652 - 0.2153i		-0.1652 - 0.2153i		-0.1652 - 0.2153i	
	-0.122		-0.122		-0.122		-0.122	
Optimized Values								
K^T	240.17	24.2736	1.3875	7.5561	13.059	4.681	-1.679	6.870
	-137.3	256.28	-158.91	153.309	-146.54	155.09	-234.17	257.016
	-195.76	84.99	-52.850	39.280	-36.04	42.31	-94.42	48.260
	36.65	0.1753	0.4948	-0.0550	0.9425	-0.0484	1.140	-0.0327
	0.162	-0.0128	0.0109	0.0069	0.013	-0.0037	0.0111	-0.0105
H/F (10^3 pounds)	20.7/1.9		19.1/0		20/0		19.9/0.0	
Mean Depth (feet)	55.00		55.00		54.99		54.92	
RMS Error (feet)	0.0414 (14%)		0.372 (90%)		0.536 (93%)		1.96 (79%)	
Maximum Error (feet)	0.175 (57%)		1.01 (65%)		1.57 (88%)		6.88 (99%)	
Eigenvalues	-17.3244		-0.8739		-1.2707		-1.0690	
	-0.7017		-0.1541 + 0.1441i		-0.2086 + 0.1628i		-0.1214 + 0.3327i	
	-0.1942 + 0.4296i		-0.1541 - 0.1441i		-0.2086 - 0.1628i		-0.1214 - 0.3327i	
	-0.1942 - 0.4296i		-0.2567		-0.2056		-0.2428	
	-0.0076		-0.0375		-0.0208		-0.0012	

Table 11. Full state feedback integral control law optimization results and performance

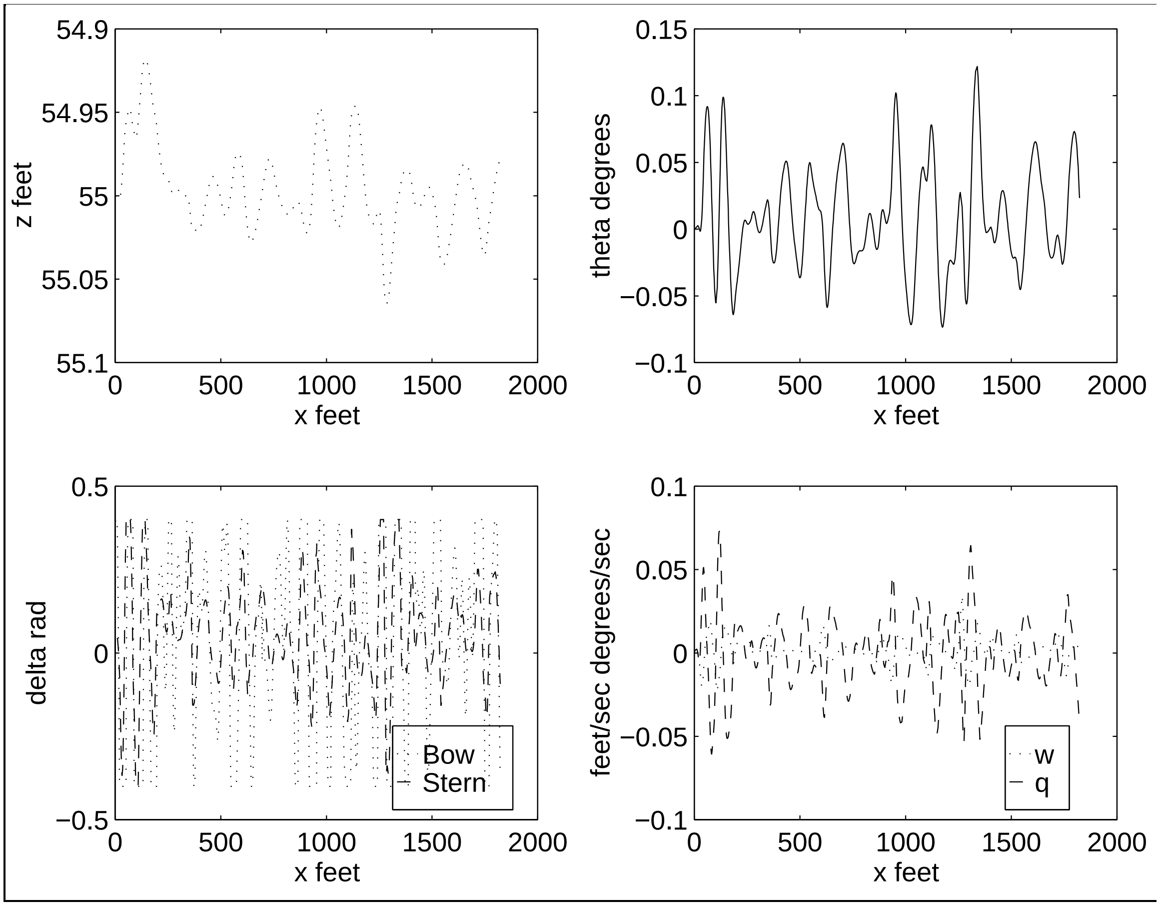

Figure 26. Optimized full state feedback with integral control simulation, sea state three (head seas)

CONCLUDING REMARKS

For each case of feedback control, the degree of control achieved generally improved by the additional state feedbacks. Full distribution of each state to both controls further reduced the error. Table 12 provides a summary of the optimizations performed, and the RMS error of each one.

Changes in the optimized trim and control law in all cases varied substantially with changes in sea state or direction. This is consistent with operational experience.

Each controller was optimized with only the goal of minimizing the mean square of the depth error. This resulted in large gains and excessive control effort. In addition, large rates of control were experienced. This would be detrimental for actual submarine operations, as there are rate limits associated with the control surfaces. These limits come from the sizing of the hydraulic plants which drive the planes, and operation concerns related to plane induced noise.

Some improvements in depthkeeping were achieved by the feedforward of the disturbance forces. This is in spite of the feedforward being based on a steady state response to a constant disturbance.

Sea State/Direction	**3/Head**	**3/Beam**	**4/Head**	**4/Beam**
Control Scheme				
Depth and Pitch Angle	0.4550	0.7549	0.657	1.23
Depth and Pitch angle with feedforward	0.102	0.556	0.810	0.883
Depth and Pitch angle with integral	0.455	0.3811	0.865	1.05
Full State with partial distribution	0.1969	0.350	0.358	0.821
Full State with partial distribution and feedforward	0.0785	0.3624	0.5171	1.25
Full State with partial distribution and integral	0.059	0.3017	0.4274	0.909
Full State	0.037	0.2638	0.2683	1.24
Full State with feedforward	0.0928	0.4121	0.400	0.792
Full State integral	0.0414	0.372	0.536	1.96

Table 12. Optimized RMS error (feet) of state feedback control schemes